Virtual Maths Camp Card Deck

Welcome to our

Virtual Maths Camp Card Deck!

First and foremost, it can be used to play standard card games. But our deck also doubles as a fun educational resource, each of its 54 cards featuring a maths game, puzzle or fact. Our hope is that it enables you to facilitate a variety of mathematical activities, virtually or in person, with or without technical support, whether you are a seasoned maths camp organizer or have little background in maths education.

Each card revolves around a mathematical problem, fact or game. Only so much can fit on a single card, so in this booklet, a much more extensive online version, as well as on the accompanying website, we provide various ways to dive deeper into each of these problems with additional content, such as explanations, extensions and background. Each card carries a unique QR code that links to the website associated with it. The online version of the booklet is available at

https://virtualmathscamp.com/card-deck.html

Our card deck also links to our interactive VMC chatbot accessible through Telegram. Once you have installed the Telegram App on your mobile device or computer and you have created an account, search for the channel

@Virtual_Mathscamp_bot

Type the code found on any of the cards (e.g. **VMC_8C**) to access an interactive version of the card content.

The 54 cards consist of four suits of 13 and two jokers. Clubs (black ♣) and diamonds (red ♦) are puzzles (non-counting-based and counting-based). Hearts (red ♥) contain fun mathematical facts. Spades (black ♠) are games, and contain basic rules and setup on the card. While the jokers do not have activities associated with them, they come with a maths joke. Some of our problems and the FEMTO game were inspired by the website https://nrich.maths.org, a great resource full of mathematical problems and games.

We hope you enjoy the deck, created by volunteers based at SAMI, AMI, AMI Ghana,

IMT, Bahir Dar University, INNODEMS, IDEMS, the Hausdorff Center for Mathematics and AIMS. We would also like to thank our supporters whose financial support has enabled us to realise this project. More information can be found on our website

virtualmathscamp.com/card-deck.html

We are constantly working on developing this resource. If you encounter any issues, or wish to give feedback, please get in touch by emailing us at

contactus@virtualmathscamp.com

Some Card Game Ideas

91 Game

Number of players: 2-7
Materials: 1 card deck for 2-3 players; 2 card decks for 4-7 players (no jokers).
Players and Cards: The game is for two to seven players, using a complete suit from a standard 52-card pack for each player plus one extra suit. Two decks are needed for four or more players. Cards rank Ace (low), 2, 3, 4, 5,

6, 7, 8, 9, 10, J, Q, K (high). As a prize, the Ace is worth 1 point, cards 2-10 face value, Jack 11, Queen 12 and King 13.

Setup: The cards are sorted into suits. One suit (traditionally diamonds) is shuffled and stacked face down as a prize pile. Each of the other players takes one complete suit.

Play: The top card of the prize pile is turned **face up** (so you see its value). Then each player selects a card from their hand with which to bid for it and places it **face down** (so you can't see its value). When all players are ready, the bid cards are revealed simultaneously, and the highest bid wins the prize card.

The bid cards are then discarded and the prize card is placed beside the player who won it. The next card of the prize pile is turned face up and players bid for it in the same way. If two players put the highest bit, the bid cards are discarded but the prize card remains on offer. A new prize card is turned face up and all players then make their next bid for the two prize cards together, then for three prize cards if there is another tie, and so on. If any of the players' last bid cards are equal, the last prize card (and any others remaining from immediately preceding tied bids) are not won by any player.

Scoring: When all players run out of bid cards, the play ends. Each player totals the value of the diamonds they have won in bids (Ace=1, 2-10 face value, J=11, Q=12, K=13) and the greater total wins the game.

FEMTO

Number of players: 2

Materials: One card of each of the following values: 2, 3, 4, 5, 6, 7, 8, 10 (8 cards total). The 8 cards are shuffled and dealt, so that each player gets 4 cards.

Play: In each round, each player puts out one card, face down (so you cannot see its value). The two cards are then turned face up. The round is won by the higher value card, unless the higher card is more than twice the value of the lower, in which case the lower card wins. E.g. 10 beats 8, 6 beats 5, 10 beats 5; but 3 and 4 beat 10… Whoever played the winning card chooses one of the two cards and puts it, face up, on the table in front of him/her (the value of this card counts as points). The player of the losing card takes the remaining card and puts it back into his/her hand. More rounds are played until one player has no cards left in their hand. The winner is the player with the greater total value of cards in front of them at the end of the game.

About the mathematics on the cards

C-D-H-S stands for Clubs (black ♣) Diamonds (red ♦), Hearts (red ♥) and Spades (black ♠). A-J-Q-K stands for Ace, Jack, Queen and King.

CA - Knockdown

Hint: What happens if you only had 5 pegs? What about 10 or 20? Can you spot the pattern?

Explanation: Start: 1 2 3 4 5 6 7 8 9 ...
1st round: ✕ 2 ✕ 4 ✕ 6 ✕ 8 ✕ ... only even numbers left
2nd round: ✕ 4 ✕ 8 ✕ 12 ... only multiples of 4 left

3rd round: ╳ 8 ╳ 16 ╳ 24 ... only multiples of 8 left
4th round: Only multiples of 16 will be left
5th round: Only multiples of 32 will be left.
32 is the only multiple of 32 under 50, so 32 is the last peg left.

C2 - OneTwoThreeFourFive

Hint: How many times does the pattern 12345 appear in the 2000 digit number?

Explanation: The five numbers 1, 2, 3, 4, and 5 add up to 15. If the pattern is continued for 2000 digits there will be 2000 ⁄ 5 = 400 sets of these five numbers. 400×15 = 6000.

C3 - Mean

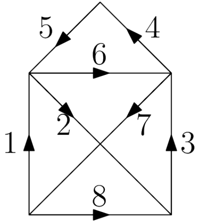
Hint: Make the first 15 (different) numbers as small as possible. What does this tell us about the 16th number?

Explanation: If the mean of 16 numbers is 16, then the total of all the numbers is 16×16 = 256. We want to make one of the numbers as big as possible, so we should make the other 15 numbers as small as possible. The integers have to be different so we should choose 1, 2, 3, ..., 15. If you add them up this gives 120 (a quick way to do it is $\frac{1}{2} \times 15 \times 16 = 120$). So the biggest number is 256 - 120 = 136.

C4 - House

Hint: Try starting from the bottom left vertex (point).

Explanation: We can draw the figure in one uninterrupted movement if we start at the bottom left (or bottom right) vertex. For example starting at the bottom left we travel along edges 1 to 8:



This kind of path is called an Euler path, and it is called an Euler circuit if it starts and ends in the same point. The degree of a vertex is the number of edges joining onto that vertex, and vertices are said to be odd or even according to whether the degree is odd or even. Euler circuits exist only in networks where there are no odd vertices, that is where all the vertices have an even number of edges ending there.

C5 - Honey Bees

Hint: How far can one bee travel with ten gallons of honey?

Explanation: 10 gallons would allow one bee to travel for 70 million miles. 70000000 ⁄ 1000 = 70000 so 70000 bees could travel 1000 miles each.

C6 - Alphanumerics

Hint: What digit must the letter R represent? If you add two 3-digit numbers together, what is the largest sum you could make?

Explanation: R must be equal to 1 since only the digit 1 can carry over from the hundreds column into the thousands column. So we now have CA1 + CAT = 1A1E.

If we look at the tens column we have A + A = 1 (or ends in 1). Since A + A is even, the only

way for it to end in the digit 1 is for there to be a carry over from the ones column.
This gives us A = 5 or A = 0. Since we know there was a carry over from the ones column, T = 9 and hence E = 0. This means A = 5 since each letter represents a different digit. Therefore the word RARE = 1510. (Though not needed, it can be verified that C = 7). Finally we have 751 + 759 = 1510.

C7 - Newspaper

Hint: Can you work backwards from 8 to work out how many sheets would be underneath this one? You might want to use bits of paper and number them to help.

Explanation: On the back of the number 8 there must be the number 7. The sheet below must have the numbers 6 and 5. Keeping going there will be 4 and 3 and finally 2 and 1. So there are three sheets underneath. Adding on 7 to the 61 gives the last page as 68. Given each sheet has four pages, there are 68 ⁄ 4 = 17 sheets.

C8 - Sword of Josephus

Hint: What happens if you had 2 pegs? What about 3, 4, 5, 6, 7, or 8 pegs? Can you spot the pattern after considering these cases?
Explanation: Starting with only 2 pegs, we know that the last peg to be knocked down would be number 1. We can work out which would be the last peg if we increase the number of pegs. To start, we record our results and look for a pattern:
2 - 1, 3 - 3, 4 - 1, 5 - 3, 6 - 5, 7 - 7, 8 - 1, 9 - 3… and so on. Each power of two (1, 2, 4, 8, …) "resets" the last peg standing to be number 1. You can see that the number of the last peg standing goes up in odd numbers after this. To work out the answer for 52, we know the last reset was the largest power of two. The largest power of two below 52 is 32, and 52 - 32 = 20. So it is the twentieth odd number that is calculated as $2\times20 + 1 = 41$.

C9 - Restaurant

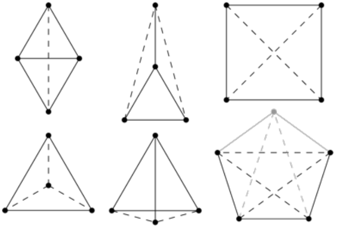
Hint: Can you work out how many legs there are at each table, if three quarters of the chairs have people on them?

Explanation: The three members of staff have six legs so take this off the total, giving 206 - 6 = 200. Now the tables, chairs and customers have 200 legs. On average, at each table there are 3 table legs, 16 chair legs and 6 customer legs (as only three quarters of the chairs are taken).This gives 3 + 16 + 6 = 25 legs per table. The number of tables is therefore 200 ⁄ 25 = 8, hence the number of chairs is 8×4 = 32.

C10 - Two Distances

Hint: Try drawing an equilateral triangle. That will give you three points that are all the same distance from each other. Where could you put the fourth point?

Explanation: If we call the two distances "short" and "long", then any such drawing of points must have a triangle with one long and two short edges (not quite obvious, but if you think about it a bit you can eliminate all other possibilities). Now where does the last point go? To each of the other 3 points, it's connected either via a long or a short edge, giving 8 possibilities. Try out each, you will notice that 2 of them do not work out, leaving 6 solutions.



Short edges are represented by **straight lines** while long edges are represented by **dashed lines**. One of these solutions can be extended to 5 points that only use two distances, see bottom right picture.

CJ - Dealing Cards

Hint: Must there be an even or an odd number of children in the circle?

Explanation: The teacher goes round the circle four times. If there was an even number of children, the sixth child would always receive a Jack each time and would have four Jacks by

the end. Since they only have two Jacks, there must be an odd number of children in the circle.
The sixth child receives a Queen in the first round because 6 is a multiple of 3. She only receives one more Queen so the answer can't be a multiple of 3.
The options are 7, 11, 13, 17, 19 … and so 7 is the fewest number of children.

CQ - Lost cards

Hint: Think about remainders when dividing. If there are two left when divided amongst five people, then the number must be two more than a multiple of 5, e.g. 7 or 32.

Explanation: As only a small number of cards have been lost, let us start by listing the numbers that work that are close to 52.
If there were no cards remaining when dealt between four people we would know that the number of cards was in the four times table, i.e …, 36, 40, 44, 48. But there are three left over, so the number of cards must be in the sequence …, 39, 43, 47, 51.

For dealing among three people with two left over we get the sequence …, 41,44,47,50
For dealing among five people with two left over we get the sequence …,32, 37,42,47. The number that appears on all three lists is 47. We could also solve the puzzle starting with the last two clues. There is the same number of cards leftover when dealing to three or five people, so the answer must be two more than a multiple of 15, so 17, 32 or 47. The only one of these numbers that fits with the first clue is 47.

CK - Monkey business

Hint: Can you work out whether number 8 will be on or off? How about number 9? What is it about the factors of these two numbers that means one of them is on and one of them is off?

Explanation: Most numbers will be pressed by an even number of monkeys, e.g. 24 has factors (1×24, 2×12, 3×8, 4×6), so monkeys 1, 2, 3, 4, 6, 8, 12, 24 will press the switch, and as this is an even number the light will be off.

Square numbers are the only ones which do not have an even number of factors, e.g. 16 has factors (1×16, 2×8, 4×4). Monkeys 1, 2, 4, 8, 16 will press the switch, and as this is an odd number that the switch has been pressed, the light will stay on (monkey 4 won't press it twice!). So we just need to work out how many square numbers there are between 1 and 1000. We find the largest square number less than 1000: 31×31 = 961, 32×32 = 1024.

So there will be 31 switches left on: switch numbers 1, 4, 9, 16, 25, 36, … , 900, 961.

DA - Table Handshakes

Hint: What is the answer if there are just 2 people, or 4 people, or 6 people. Can you find a way of making the cases with fewer people help you count the cases with more people?
Explanation: For the case with 2 people there is just 1 way. For 4 people there are 2 ways, one of which is

For 6 people, we can consider the options for who number 1 shakes hands with. Number 1 cannot shake hands with number 3 or 5 as that would leave someone cut off from the group. If number 1 shakes hands with number 2, then we know that the other four people can shake hands in 2 different ways (using our result from before).

Number 1 could shake hands with number 6 and there would also be 2 different ways for the other four to shake hands. Finally number 1 could shake hands with number 4, and that would just leave one option. So the result for 6 people is:

$$2 + 1 + 2 = 5.$$

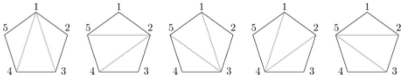
For eight people number 1 could shake hands with number 2 (leaving a group of 6 people which we now know there are 5 ways to shake hands), number 4 (leaving a group of 4, so 2 ways) number 6 (leaving a group of 4, so 2 ways) or number 6 (leaving a group of 6 so 5 ways). So the answer for eight people is

$$5 + 2 + 2 + 5 = 14.$$

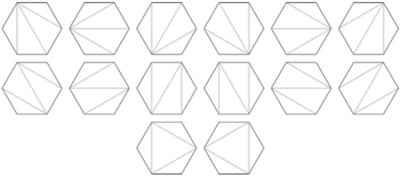
D2 - Triangular Slices

Hint: In how many ways could you do it if the cake were a square? What if it were a five-sided pentagon? Can you use the symmetries of your cake to count faster?

Explanation: It is best to start with a simpler question and look for patterns. Start by looking at a square; it is easy to see that there are two different ways we could slice the cake. For a pentagon, there are five ways:



Finally for a hexagon there are 14 ways:



In this diagram you can see the solution is found by thinking of 6 + 3 + 2 + 3 = 14, but you can arrive at the solution by doing 5 + 2 + 2 + 5 = 14 where 5 is the answer for a pentagon and 2 is the answer for a square. The equivalent combination for the pentagon is 2 + 1 + 2 = 5, where 1 is the result of the triangle (which has of course only one option) and 2 is the result for a square.

D3 - Scoring Goals

Hint: Imagine the sequence of all goals and think about the positions in the sequence in which the second football team can have scored their 2 goals. Try to find the answer in the cases the score was 3 - 2 or 4 - 2.

Explanation: Imagine the score was only 3 - 2. Let us think about three goals scored by the first (home) team (HHH) and two goals scored by the second (away) team (AA). We could try and list the possibilities. e.g. AAHHH would represent the first two goals scored by the away team and then the next three goals scored by the home team.

All the options for the five goals are : AAHHH, AHAHH, AHHAH, AHHHA, HAAHH, HAHAH, HAHHA, HHAAH, HHAHA, HHHAA. There are

10 options in total. A way to calculate this is to think about there being 5 options for one away goal: _ _ _ it could be in one of any of the 5 positions (e.g. A _ _) . The other away goal could be in one of the remaining 4 positions. So you would think there would be 5×4 = 20 ways of choosing where the two goals would go. However, we have double counted, because we have included every possibility (e.g. AAHHH) twice (one goal in position 1 and the other in position 2 but also the other way around). The order in which we selected the 2 goals of the second team does not matter: we need to divide by 2. So the answer if the score was 5 - 2 is 5×4 ⁄ 2 = 10. For the question of how many ways can the goals be scored if the score is 5 - 2, we can use the same logic. There are 7 possibilities to select the position for one goal; then there are 7 - 1 = 6 possibilities to select the position of the other goal ("-1" because one position has already been selected), therefore you get 7×6 = 42. However, the order in which you selected the 2 goals of the away team does

not matter: you need to divide by 2. So we get $7 \times 6 / 2 = 21$.

D4 - Colouring

Hint: Think about how many different colours could go in the first section, then the second ...
Explanation: There is a choice of 5 colours for the first section. Once a colour has been chosen, the second section cannot be the same. So there are only four colours to choose from for the second section. For the third section, there are four colours to choose from, as it cannot be the same colour as the second one yet it can be the same colour as the first one. Thus, the number of tricolour flags with 5 colours to choose from are $5 \times 4 \times 4 = 80$.

D5 - Avoid the River

Hint: In how many ways you can go from home to school in a 2 by 2 grid? What about in a 3 by 3 grid?

Explanation: This is not an easy combinatorial problem. It is better to start with a simpler version and familiarise with the problem. It is indeed intuitive to find the result for 2 by 2 grid: there are indeed only 2 possible ways. As far as the 3 by 3 case is concerned, students might easily draw all 5 ways explicitly. However, when it comes to the 4 by 4 case, it is easy to lose the count. Therefore the problem aims at making students think how to tackle this counting in a more systematic way. In particular, students should think whether they can decompose their counting in a way they can employ results known for a lower number of people. They may notice for example that the result for a 3 by 3 grid is: 2 + 1 + 2 = 5, 2 is the result for a 2 by 2 grid. Moreover, the result for a 4 by 4 grid is 5 + 2 + 2 + 5 = 14, where 2 is the result for a 2 by 2 and 5 is the result for a 3 by 3 grid.

D6 - Shaking Hands

Hint: Start with a small number of children, e.g. 3, 4, or 5. What is the number of handshakes you have to add if another child joins the party?

Explanation: By counting from small examples, you may realise that each time you

add a person at the birthday party with n children, there are n more handshakes.
If we have 2 children (say A and B) we just have 1 handshake AB.
When we have 3 children (say A, B, and C) we have 3 handshakes: AB, AC, and BC. There is the original one and 2 extra ones now that C has joined, giving us $1 + 2 = 3$.
If we add another child (say D) they must shake hands with A, B, and C and all the other handshakes still take place. So there will be $1 + 2 + 3 = 6$ handshakes. If we add another child to this group of 4, then there will be another 4 handshakes. So $1 + 2 + 3 + 4 = 10$ handshakes.
If we follow this pattern we can find the answer for 8 children is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28.$$

Another way to think about it is if you are one of the 8 children, you need to shake hands with the 7 other people. So does every other child at the party. Since there are 8 of you, and you each shake hands with 7 other people, there should be 8×7 handshakes. However, as it takes 2 people to shake hands we have counted each handshake twice. So we divide by two and the answer is $8 \times 7 / 2 = 28$.

D7 - Pieces of Cake

Hint: Start with a small number of cuts, e.g. 1, 2, 3. How can you add another cut to get the biggest number of pieces?
Remember it doesn't say that the pieces have to be the same size! What happens if many cuts pass through the same point?
Explanation: With one cut you always get 2 pieces. With two cuts you can get 4 pieces. With three cuts, if all cuts pass through the same point you get 6 pieces. However, you can get 7 pieces if the third cut doesn't pass through the same point as the previous two. Use this idea to get the maximum number with four cuts. So to get the most pieces, make sure that no more than two cuts pass through the same point, otherwise you get less pieces. With four cuts you can get 11 pieces. Do the same again for five cuts, start with the four cuts that give 7 pieces and add a cut without passing through any meeting

points (intersections) and you can get 16 pieces.

D8 - Mixed-up Socks

Hint: Start with matching one of the red with one of the green, and the other red with a blue. You could write this as RB, RY. What would the other two pairs have to be?

Explanation: You can solve this problem by listing all the options. You could have RB, RY, YG, BG or RB, RG, GY, BY or RY, RG, YB, GB. You could also solve it by thinking that, starting with the red socks, there are three different colours that the two socks could be matched with. If you choose RB and RY as the first two pairs, you can't then match BY together because that would leave GG for the last pair which isn't allowed.

So once you have picked the two colours to go with red, there are no more choices to make. There are 3 ways to pick two colours from a choice of three, so the answer is 3.

D9 - Counting Squares

Hint: Can you start with a 4 by 4 square and work out how many squares there are?

Remember to keep track of how many 1 by 1, 2 by 2, 3 by 3 and 4 by 4 squares you find.

Explanation: There is one 10 by 10 square, four 9 by 9 squares, nine 8 by 8 squares, sixteen 7 by 7 squares .... one hundred 1 by 1 squares, all of which are square numbers. So we need to add all the square numbers from 1 to 100: 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = 385.

D10 - Going to School

Hint: What is the total number of steps you have to make? Think about when you could make the two steps north. You could first try to find the answer if the grid was 2 by 3 and 2 by 4.

Explanation: First of all, we can realise that, in order to get to school, you need to do a total of 2 steps north and 5 steps east. If we think about the sequence of 2 + 5 = 7 steps (S1, S2, S3, S4, S5, S6, and S7), then the problem is equivalent to counting how many ways you select 2 objects (the positions of the two steps towards north) from a total of 7 objects (the total number of positions of the steps). There are 7 possibilities to select the position for the first step towards north; then there are 7 - 1 = 6 possibilities to select the position of the second step towards north ("-1" because one position has already been selected), therefore you get 6×7 = 42. However, the order in which you selected the 2 positions of the steps towards north does not matter: you need to divide by 2, to give the final answer of 21.

DJ - Different Paths

Hint: Think about how many options you have to select the first path, then how many you have for the second and so on. Be careful that you cannot choose the same path twice in a week.

Explanation: You have 5 possible days of the week and you would like to have one different path per day. On the first day, you have a choice of 5 paths. On the second day, you only have 4 paths left to choose from, because you can't take the one you took on Monday. On the third day, you have 3 to choose from, and so on. The solution is therefore 5×4×3×2×1 =120. This is counting how many ways you can

order 5 objects (paths) in 5 positions (days of the week). This is a problem on *permutations* (without repetition).
The mathematical symbol ! is called **Factorial.** It represents the operation of multiplying by decreasing integers until you reach 1. So $5\times4\times3\times2\times1$ can be written as $5!$

DQ - Bar Stools

Hint: You could have five students sitting on the five chairs. You could have two teachers sitting on chairs number 2 and 4 and students in chairs 1, 3, and 5. Can you think of any other options? Can you think of the maximum number of teachers that could sit on the chairs?
You might like to start with a smaller number of chairs and work your way up.
Explanation: Think of chairs as having a teacher (x) or a student (o).
For 5 chairs we can have
ooooo, xoooo, oxooo, ooxoo, oooxo, oooox, xoxoo, xooxo, xooox, oxoxo, oxoox, ooxox, xoxox
Total = 13

There is a way we can build up to this solution by thinking about the answer if there was 1 chair, 2 chairs, 3 chairs etc.
1 chair has 2 options: x , o
2 chairs has 3 options: ox, oo, xo
3 chairs has 5 options: oox, ooo, oxo, xox, xoo
Each time you add a chair you can generate the new options by simply adding a student to the start of all the previous options. You can also add a teacher and then a student to all of the options from two chairs ago. Let's see this with 4 chairs. Add o onto all the 3 chair options gives: ooox, oooo, ooxo, oxox, oxoo
And adding xo onto all the 2 chair options gives: xoox, xooo, xoxo
This gives 5 + 3 = 8 options in total.
For the 5 chairs we can add o onto all the 4 chairs options and and xo onto all the 3 chair options which gives 8 + 5 = 13 as before.

DK - Paving Paths

Hint: In how many different ways can you do that in case you had fewer paving slabs for a shorter path? Think about a good strategy to classify different ways. For example, how many ways can you have 2 pairs of slabs lengthwise?

Explanation: This problem is firstly meant to make you think about a good strategy to count. You can draw all possibilities for shorter paths. However, this will soon become too lengthy. A first step towards finding an organizing principle could be classifying the paths accordingly to how many pairs of slabs placed lengthwise there are. Let us denote as n the number of paving slabs. For n = 1, there is clearly only one possibility (no pairs); for n = 2 there are 2 possibilities (no pairs, and 1 pair). For n = 3 there are 1×(no pairs) + 2×(1 pair) = 3 ways; for n = 4 there are 1×(no pairs) + 3×(1 pair) + 1×(2 pairs)= 5. Using this strategy we can easily count n = 6: there are 1×(no pairs) + 5×(1 pair) + 6×(2 pairs) + 1×(3 pairs) = 13. Looking at the sequence 1, 2, 3, 5, 8, 13,… you might recognize the pattern!

HA - Birthday Probability

Hint: The probability that two people have the same birthday is just 1 minus the probability of everyone's birthday being different. You should ask yourself the following questions: If we only have 2 people, what is the probability their birthday is different? If we had 5 people, in how many ways can we select two to compare their birthdays? What do we get if we use the same approach for 23 people?

Explanation: The probability of any 2 people having different birthdays is roughly 364 ⁄ 365. There are 253 different pairs of people in a room of 23 people, so the probability of them all having different birthdays is roughly 364 ⁄ 365 to the power 253. This is 0.4995, slightly less than 50%, so the probability of having 2 people in the room with the same birthday is just over 50%. Actually we should be including leap years but the result is still true if you do.

H2 - Kaprekar's Number

Hint: For example, if you start with 4252, you should do the calculation 5422 - 2245 = 3177. Then do 7731 - 1377 and keep going.

Explanation: You always end up with the number 6174. This is known as Kaprekar's number. For example 5432 – 2345 = 3087, 8730 – 0378 = 8352, 8532 – 2358 = 6174, 7641 – 1467 = 6174.

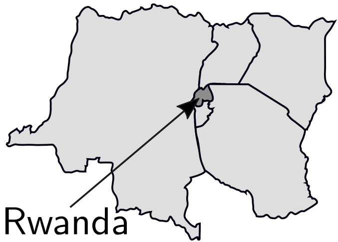
H3 - Palindrome Cube

Hint: Take the number 1030301. It is a palindrome. It's cube root is 101, which is also a palindrome. Can you find other examples that are both palindromes? (Remember that all one digit numbers are palindromes!)
Explanation: According to the On-line Encyclopedia of Integer Sequences, The sequence of positive integers whose cube is palindromic begins:
1, 2, 7, 11, 101, 111, 1001, 2201, 10001, 10101…
The corresponding sequence of palindromic cubes begins:
1, 8, 343, 1331, 1030301, 1367631, 1003003001, 10662526601, 1000300030001, …
2201 is the smallest (and only one known so far) which is itself non-palindromic.

H4 - Four Colours

Hint: Can you colour a map of Africa with five colours so that no two countries that touch are filled in with the same colour? Can you also succeed with four colours? How about three colours?

Explanation: The Four Color Theorem says that given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colours are required to colour the regions of the map so that no two adjacent regions have the same colour. One can see why it is not possible to do this with three colours by looking at the situation around Rwanda (try this yourself!):



The proof that only four colours are needed is very tricky.

H5 - Ramanujan

Hint: 72 can be expressed as $2^3 + 4^3$ but it is the only way with two positive cubes. Can you find the two ways to make 1729?
Explanation: $1^3 + 12^3 = 1 + 1728 = 1729$ and also $9^3 + 10^3 = 729 + 1000 = 1729$.

H6 - Abundant Numbers

Hint: You could make a 10 by 10 grid of the numbers 1 to 100 and colour in the abundant numbers.
Explanation: The next abundant number is 20, because the proper divisors of 20 are 1,2,4,5, and 10 which add up to 22. Can you explain why prime numbers can never be abundant?

H7 - Folding Paper

Hint: Let us say that the paper thickness is about 0.1 millimeter (or 0.0039 inch). If you fold it once on itself, how thick would it be? What about it you fold it once more on itself? What about if you fold it 10 times?
Explanation: The distance between the Earth and the Moon is a bit less than 400000 kilometers (or 250000 miles). For us the paper

is 0.1 millimeter (mm) thick. If we fold the paper once, its thickness would be doubled, so it becomes 0.2mm. Folding it again gives 0.4mm. If we fold it 10 times, it becomes $(0.1\text{mm})\times 2^{10}$ (two to the power 10). Now, if we compute $(0.1\text{mm})\times 2^{42}$, we get 440 000km!

H8 - Prime Removal

Hint: If you take away the last digit the number would be 7393913, which is a prime number. Keep going ...

Explanation: A right-truncatable prime is a prime number that remains a prime when you successively remove the last digit. We can look for the biggest right-truncatable prime by starting with the one digit primes 2, 3, 5, and 7. Then try to add another digit to the right of these so that the new numbers are also prime. We could have 23, 29, 31, 37, 53, 59, 71, 73, and 79. Now we try to add another digit. We could have 233, 239, 293, 311, 313, 317, 373, 379, 593, 599, 719, 733, 739, and 797.
If we keep going with this process we end up with these eight digit numbers 23399339, 29399999, 37337999, 59393339, and

73939133. There are no nine digit prime numbers that start with any of these eight digits so the biggest right-truncatable number is 73939133.

H9 - Shuffling Cards

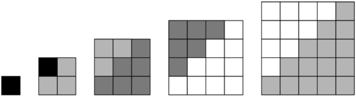
Hint: How many arrangements of the 52 cards can we have? Maybe this is too big to start with. Let's try out a smaller number of 3 cards, how many different arrangements can we have? Now try 4 cards?
Can we now use this to get to the 52 cards?
Explanation: We want to work out how many different combinations of 52 cards there are. Let us start with thinking about just 3 cards. We could compare this to thinking about the number of ways 3 people could sit in three numbered chairs. Any of the three people could sit on the first chair, then either of the 2 remaining people could sit on the second chair and then there is 1 person left for the last one. We can multiply the number of choices for each chair ($3×2×1 = 6$) to find how many different ways there are for the 3 people to be arranged. Note that the pattern here is taking

the initial number then multiplying consecutively by the next smallest integer until you reach 1. This process has a name, it is called **Factorial.** The symbol for it is !. e.g. 3! = 6. Generally if you want to find the number of different ways you can rearrange n items, you can find n! = n!×(n - 1)!×(n - 2)!×…×1 The number of ways to shuffle 52 cards is 52! = 52×51×50×…×3×2×1, which is roughly 8×10^{67}.

H10 - Square numbers and triangular numbers

Hint: Triangular numbers can be drawn with dots in the shape of a triangle. Square numbers can be drawn with dots in the shape of a square grid. See if you can work this one out visually first. Can you express it algebraically as well?

Explanation: We first show this visually, in the image below you can see that 4 = 1 + 3, 9 = 3 + 6, 16 = 6 + 10, and 25 = 10 + 15.



If we take T = triangular and S = square, then we can draw the S-number as an S-grid of points, the smaller T-number forms the upper T of points, the larger T-number forms the lower T AND the diagonal of points. These are consecutive triangular numbers making a square. The side length in points of the bigger T-number is the square root of the S-number.

HJ - Benford's Law

Hint: Have a look at some data sets that span many orders of magnitude. For example, what are the first digits of: the first 100 prime numbers (or more!), the populations of the countries in Africa, the heights of the tallest buildings in the world?

Explanation: Many data sets (for example, populations of countries, heights of buildings, stock prices) or famous sequences (Fibonacci numbers, prime numbers) are spread over very different scales (orders of magnitude). Many of these big sets of data arise from decreasing power laws. Specific properties of the latter lead to Benford's law.

HQ - Lychrel Numbers

Hint: Pick a positive integer (whole number), e.g. 57. Reverse the digits to get 75. Adding gives 57 +75 = 132. Then reverse this new number and add together: 132 + 231 = 353. This is a palindrome as it reads the same forwards as backwards, so we stop. Can you find a number that takes longer to reach a palindrome?

Explanation: Some numbers end up in a palindrome after one step, e.g. 243: 243 +342 = 585. Some take a bit longer, e.g. 87:
87 + 78 = 165, 165 + 561 = 726,
726 + 627 = 1353, 1353 + 3531 = 4884.
Some numbers take a very long time – 89 takes an unusually large 24 steps (the most of any number under 10,000) to reach the palindrome 8813200023188. A number is called a Lychrel number if it never reaches a palindrome. No one has proved that a Lychrel number exists. One possibility is 196 - the first billion steps have been carried out by computers and it hasn't reached a palin-drome… but this does not prove it never will!

HK - Collatz Conjecture

Hint: For example, if you choose the number 6 you would get this sequence 6, 3, 10, 5, 16, 8, 4, 2, 1. So it took 8 steps to get to 1. Try some numbers of your own.

Explanation: It is easy to understand the rules and start generating sequences, but no one knows whether all numbers end up at 1; this is an unsolved problem in mathematics.

The longest sequence for a number smaller than 100 is for 97, which takes 118 steps but finally gets to 1. The number just before, 96, only takes 12 steps! Using computers, all numbers up to roughly 2.95×10^{20} have been checked and they all go to 1.

SA - Fifteen

Further instructions: Remember you can't use two numbers to make 15, you can't use four numbers, it must be three numbers. You can have more than three cards in your hand and use only 3 of them. The game is a draw if all the cards are gone and nobody can make 15.

Strategy Tips: Write down all the combinations of three numbers that add up to

15. How can you be sure that you have found all the combinations? Which numbers appear in 2, 3, and 4 of these combinations?

Think about other games you might have played that this is similar to. Is it an advantage to go first or second?

S2 - Patience

Further instructions: Remove the Jokers before starting. A has a value of 1 in this game. If you manage to play all your cards, the last stage of the game is to pick up pairs of piles of cards whose top cards add up to 11, or three piles whose top cards are J, Q, K. You have won the game if you can pick up all the cards. If you can't put down all your cards, count how many are left.

Strategy Tips: This game is mostly luck - but you need to think carefully about how many J, Q and K you cover to win the final stage!

S3 - Twenty-one

Further instructions: An example game.
Player 1: 1,2,3 Player 2: 4
Player 1: 5,6 Player 2: 7,8,9
Player 1: 10 Player 2: 11,12,13

Player 1: 14,15,16 Player 2:17,18,19
Player 1: 20 Player 2: 21
Player 1 wins!

Strategy Tips: The person who says 20 wins, as they force the next person to say 21. How can you make sure you say 20? Try to work back-wards. After you have played several times, you might work out who is going to win before you reach 20 - think about how you know this. Can you work out if it is better to go first or second? Can you make sure you always win?

S4 - Nim

Further instructions: You can remove as many sticks as you like each turn as long as they come from the same row. For example an opening move could be to take all 5 sticks from the bottom row. You must take at least one stick each turn. You are trying to leave just one stick left for your opponent to be forced to take and therefore lose the game.

Strategy Tips: Try to think a few moves ahead. For example, if you can remove a stick to leave the pattern

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| |

then you will be able to win. Try to work out if it is better to go first or second.

S5 - Mastermind

Further instructions: Here is how a game might work. Player 1 chooses 1) 4 of Hearts, 2) 3 of Clubs, 3) 8 of Spades, 4) 10 of Spades and places them face down. Player 2 lays down for example: 1) 6 of Hearts, 2) 4 of Spades, 3) J of Diamonds, 4) 3 of Spades. Player 2 has three correct suits (Heart, Spade and Spade) and two in the correct position - 1) and 4).
So Player 1 would say "you have two in the correct position and one in the wrong position", but doesn't say which ones - an easy way to record this would be xxo. Then player 2 would guess again. They might think that it was the last two cards that were perfectly correct, so keep these and guess a different first two. Record the results on pen and paper like this:
HSDS xxo
CCDS xoo

Note that it is just the suits we are interested in, not the numbers. Player 1 is not saying which ones are correct, just saying how many are correct. Player 2 keeps guessing until all four are correct.

S6 - Latin Squares

Further instructions: This could be a one player puzzle or get a group of people to try it together as it is quite tricky!
Strategy Tips: Start by ignoring the suits, and try to get the ranks in a square so each rank appears only once in every row and column.

S7 - Gomoku

Further instructions: This game is traditionally played with black and white counters, but you could use any different counters, or little pieces of paper. You could also draw the grid in pen and then use pencil to mark x or o for each player and then rub them out when you are finished to be able to use the grid again.
Strategy Tips: It is very important to be watching your opponents' moves as well as trying to make a line of 5 yourself. If your opponent has three in a row with nothing at

either end - this is called "open 3" - then it is a good idea to use your turn to block one end. If they manage to get "open 4" you have certainly lost!

A good attacking strategy might be to aim for two overlapping lines of open 3s - like a cross - so that it is hard for your opponent to defend.

S8 - Sim

Further instructions: You might want to draw all the possible lines between the points to start with and then colour over them. This would make the game a bit easier to play. If you don't have two different colours you could use solid and dotted lines for the two players.

Strategy Tips: Try to make moves in a way that removes options from your opponent. You might want to try and complete triangles that are of mixed colour.

S9 - Pong Hau K'i

Further instructions: If the same moves are repeated three times, you can decide the game is tied. Each time you play, you should change who goes first or swap starting

positions (if you were in the top two positions the first time, take the bottom two next time).
Strategy Tips: Think about whether it is better to go first or second? What is the position you must achieve so as to block the other player?

S10 - Stop or Dare

Further instructions: If the cards are all turned over before the target is reached, just reshuffle the pack and continue.
Strategy Tips: Try and keep count of how many K and A cards have come up already. This could change how daring you are feeling. If someone else is close to 100, it might be best to turn over more cards than you would normally do.

SJ - Twenty-four

Further instructions: This is a game for two or more players. A few combinations might be impossible. In this case, put the cards back and shuffle again to choose another 4.
Strategy Tips: Think of the factors of 24 to cover i.e, 1×24, 2×12, 3×8, and 4×6. If you have either of the factors in your list, say 1, 2, 3, 4, 6, 8, … use the remaining numbers to

make the other number, or use your numbers to create combinations.

Strategy Tips: Think about where it is going to be best to place the biggest numbers.

SQ - Card Sums

Further instructions: It might help if everyone has pen and paper. After one minute, everyone says their highest total. The person who says the highest number explains how they came to that number, by arranging the cards. If they are right, they win a point. If incorrect, the person with the next highest total attempts to justify their answer. If two people have the same answer, they get a point each.

SK - WinTwoThreeFour

Further instructions: You could play a certain number of rounds or play until a player gets to a certain score, e.g. 25.

Strategy Tips: It is tempting to focus on making your 4 digit number as big as possible, but maybe everyone else is trying to do this too! It is important to watch how your opponents are playing and try and adapt your strategy to beat them.